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Structural analysis is a systematic approach to curriculum development representing an attempt to organize terminal performance objectives for a unit of subject matter into a sequence of prerequisite competencies which must be satisfactorily mastered if successful terminal performance is to occur. The technique involves asking the question: "What competencies must a person already possess in order to obtain a satisfactory performance level on some specified objective, given no instruction beyond those definitions specific to the objective in question?" By asking this question of all identified competencies, a hierarchy of requisite competencies is generated which parallels the learning process appropriate to the final task. In preparing instructional materials, the hierarchy provides the developer with a guide of what to cover and the sequence to follow. The following progression is forwarded for preparing instructional materials appropriate for a single specific competency: (1) goal statement, (2) definitions, (3) recall of prior knowledge, (4) integration, (5) demonstration, and (6) practice. Such an arrangement of competencies based on their contingent relationship further requires the consistent use of behavioral statements in the description of desired-end states and their prerequisite competencies. (DM)

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STRUCTURAL ANALYSIS AS
AN AID TO CURRICULUM
DEVELOPMENT

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The purpose of this paper is to present a rationale for the use of a specific analytic technique for purposes of curriculum development which I have chosen to call structural analysis, and to further illustrate the use of this technique in sufficient detail so that it may be adopted by others. Structural analysis refers to a process by which ultimate curriculum objectives, stated in behavioral terms, are sequentially analyzed to identify and specify each of the competencies which students must acquire if the terminal objectives are to be achieved. Structural analysis leads to the identification of not only the competencies themselves, but the arrangement or sequence in which the competencies must be arranged to make progression possible. A "picture" specifying the competencies and their relation to one another is termed a hierarchy of requisite competencies or "hierarchy" for short. Once such a hierarchy has been developed for an instructional unit, the curriculum developer can then use it as a map to guide him in the preparation of materials. If the hierarchy truly recreates the internal logic of the subject matter, instructional materials based on it can be expected to maximize learning.

This paper will discuss both the concept and use of the hierarchy in curriculum development, and present guidelines as to the application of this technique to the development of instructional materials. Emphasis will be placed both on the rationale for the use of the technique advocated, and the manner in which the technique is to be applied.

The concept of the hierarchy as amplified on in this paper, as well as others among the ideas presented, are not unique to this author. Credit for the initial formulation of many of these ideas goes to

Robert M. Gagne. Sources such as Gagne (1962, 1965a, 1965b, 1965c, 1966a, 1966b), Gagne et al. (1965), Gagne and Bassler (1963), Gagne and Paradise (1961), and Gagne, Mayor, Garstens, and Paradise (1962) may be consulted for background reference. In this paper an attempt will be made to integrate and reemphasize many of Gagne's ideas, to amplify and expand upon some of them, as well as show their application.¹ It is felt that in today's world with its great potential for change, and in the world of education in particular wherein a curriculum revolution of sorts seems to be in progress, it would be useful to expound upon and illustrate the application of a set of principles having significant import for the development of instructional materials.

The Concept of the Structural Hierarchy

The purpose of structural analysis is to identify all the competencies upon which some final task performance is based, and to determine the relationship between these competencies in terms of contingencies. These competencies are then arranged in a hierarchical picture in which contingencies are displayed by placing competencies higher or lower in the picture and by drawing arrows between them as appropriate. The hierarchy which results from this process is meant to mirror the internal logic of the process which must take place within the learner's head in order that he can proceed from having no competency in a subject matter area to having sufficient competence to master a stated final objective.

¹The author completed a portion of his graduate studies under Dr. Gagne and has been in the process of using and extending many of his concepts over the course of the past five years.

Thus, the hierarchy is more than the arbitrary tool of the psychologist, behavioral technologist, or curriculum developer. It is more than just an outline. It is a schematic diagram of the competencies which must, of logical necessity, be acquired en route to a specified end. It is, if you will, a picture of the required learning activity if a specified end point is to be reached. Obviously, if one has developed such a picture, one can then develop instructional materials which follow it, thus enabling the instructional material to parallel the required learning activity as closely as possible. Thus, the hierarchy has conceptual value as well as practical value; in fact, its conceptual value, that of paralleling the required learning activity, is what enables it to have practical value.

How to Develop a Structural Hierarchy

STEP 1. Specify the terminal objective or objectives of an instructional unit in behavioral terms. (The reason why behavioral terms are necessary will be dealt with below.) In effect, you are to start out where you want to end up by specifying the desired end point. The determination of this end point may be judgmental in a few respects. First, within a limited area of knowledge, experts may have difficulty in agreeing about a satisfactory terminus. Second, there is inevitable confusion about the size of the "chunk" that is to be considered. Does one work with all knowledge in a subject area or only a unit at a time? The hierarchical approach has no definitive answer to either issue. The unit approach is recommended, however, and experts must come to some

agreement on terminal objectives for a unit before the analysis can begin.

Some examples of terminal objectives appear below:

From a unit on Fractions (Tuckman, 1968):

Finding single numerical values for expressions containing dissimilar fractional expressions in sequence (either taken from drawing dimensions or stated directly) requiring addition, subtraction, multiplication, and division.

From a sequence on Classification in Science (AAAS, 1965):

Constructing a multistage classification schema given a collection of objects, plants, or animals, which enables someone else to identify each object in the collection.

From a unit on "New" Mathematics (Gagne, Mayor, Garstens, and Paradise, 1962):

Stating, using specific numbers, the series of steps necessary to formulate a definition of addition of integers, using whatever properties are needed, assuming those not previously established.

Each of the above three examples represents a desired terminal performance by students in a particular area of subject matter. In the case of the first example, the statement given represents a complete specification of terminal performance. In the second example, a second objective: "Identifying the slope of linear graphs through the origin", was also specified. In the third example, a second objective: "Adding integers", was also specified. Once the terminal objective or objectives for a unit have been formulated, structural analysis can proceed.

STEP 2. For each of the terminal performance objectives ask the question: What competencies, stated in behavioral terms, must students

acquire in order that they can satisfactorily complete the terminal objective given nothing but additional definitions specific to the final objective? The answer to this question will take the form of one or more competencies, stated behaviorally, as prerequisite to each terminal objective. It can then be assumed that if a student were to have acquired the prerequisite competencies, he could by being given instruction specific to the final objective (and not necessary for the mastery of the prerequisite competencies) perform adequately on the final objective.

Again, a few examples would be helpful. (Refer to the three hierarchies in the appendix: the first for Fractions (Tuckman, 1968), the second for Classification in Science (AAAS, 1965), and the third for Adding in the New Mathematics (Gagne, Mayor, Garstens, and Paradise, 1962). In the hierarchy on Fractions, when one asks what a student must know to solve fractional expressions requiring adding, subtracting, multiplying, and dividing, the answer is that he must be able to solve sequences requiring adding and subtracting on the one hand and sequences requiring multiplying and dividing on the other. (These are labelled as Ia and Ib in the hierarchy.) Beyond the attainment of these two broad competencies, only the instruction indicating that these competencies are to be integrated would be required to enable the learner to satisfactorily attain the terminal objective.

Consider the Classification hierarchy. In order for a student to be able to construct a multistage classification schema (one of the terminal objectives), he must first have attained the competency to construct and use a two-dimensional punch card classification system, construct and use

a two-stage color-coding classification system, and construct and use alternative classification schemas for the same objects. Thus, the general objective of constructing a classification schema requires the competency to construct and use two different schemas on the same occasion. By being instructed to integrate these three prerequisite competencies, the final objective would be reached.

Finally, in the New Mathematical approach to adding integers, the final task of formulating a definition of the addition of integers requires the prerequisite attainment of competence in "supplying the steps and identifying the properties assumed in asserting the truth of statements involving the addition of integers" and "stating and using the definition of the sum of two integers, if at least one addend is a negative integer".

Thus, the final task objective or objectives are analyzed into their prerequisite competencies by asking what competencies a person must have acquired to attain satisfactory final task performance given no additional instruction beyond definitions specific to the final task.

STEP 3. Analyze each of the competencies identified in Step 2 by asking what competencies a student must attain as prerequisites for these, given nothing in addition but specific definitions. Thus, the second step is repeated on those competencies found in Step 2 to be prerequisite to the final objective. In the Fractions hierarchy, adding and subtracting fractions has four prerequisite competencies which include adding two dissimilar fractions, subtracting two dissimilar fractions,

expressing mixed numbers as improper fractions and expressing improper fractions as mixed numbers.

In the Classification hierarchy, constructing and using a two-stage color-coding classification schema has as prerequisites competencies dealing with the ordering of colors, classification of common objects differing in many respects, and classification of aquarium objects.

STEP 4. Analyze the competencies identified in Step 3 into those prerequisite competencies that must be attained prior to the attainment of those in STEP 3. In other words, the procedure utilized in Steps 2 and 3 is repeated in Step 4.

STEP 5. Repeat Step 4 until a point is reached at which analysis yields competencies that are no longer reducible. (In fact, it is possible to reduce them once more into the underlying components of "intelligence" upon which the lowest level competency is based [Gagne and Paradise, 1961]).

The net result of the structural analysis is the production of a hierarchy such as those shown in the Appendix. As you can see, the final performance objective appears as the top of the hierarchy (in two of the three cases labelled as "task"). Beneath the final objectives or tasks are a series of statements (in behavioral terms) enclosed in boxes and connected to one another (often in complicated patterns) by arrows. The items in each box are competencies, identified using the five steps described above.

The competencies in the hierarchy are further identified by level using Roman numerals. Typically, the levels are numbered from the top down (although the reverse has been done in the Classification hierarchy). Typically also, the final objective does not have a level number. The levels correspond to steps on the complexity-simplicity continuum. The greater the complexity of a competency, the higher it belongs in the hierarchy. In carrying out the steps in the analysis, occasionally a level is reached where a competency has no prerequisites, but prerequisites are identified in following steps at lower levels.

The determination of the level of a competency is judgmental. Examples can be drawn from the Fractions hierarchy. One of the three competencies that "feeds" directly into the final task (in addition to Ia and Ib) is IIIf ("stating sums and differences in length as fractional problems"). IIIf is no less a prerequisite than Ia or Ib; however, it is less complex and is therefore assigned to a lower level in the hierarchy. One useful guidepost in assigning a competency to a level is the number of prerequisites it has. Ia and Ib each have prerequisites at four different levels while IIIf has prerequisites at only one level. Notice, too, in the Fractions hierarchy that competencies IIa, b, c, and d each have prerequisites at level IIA while competencies IIe, and f have no prerequisites at this level. This judgment is based on the determination that competencies IIe and f's most immediate prerequisite is no more complex than those prerequisites twice removed from competencies IIa, b, c, and d.

The process of structural analysis of a subject matter unit into

requisite competencies at appropriate levels is one in which someone can expect to have difficulty at first but improve with practice. Once mastered, however, skill in structural analysis is highly useful for curriculum development.²

Why Behavioral Objectives?

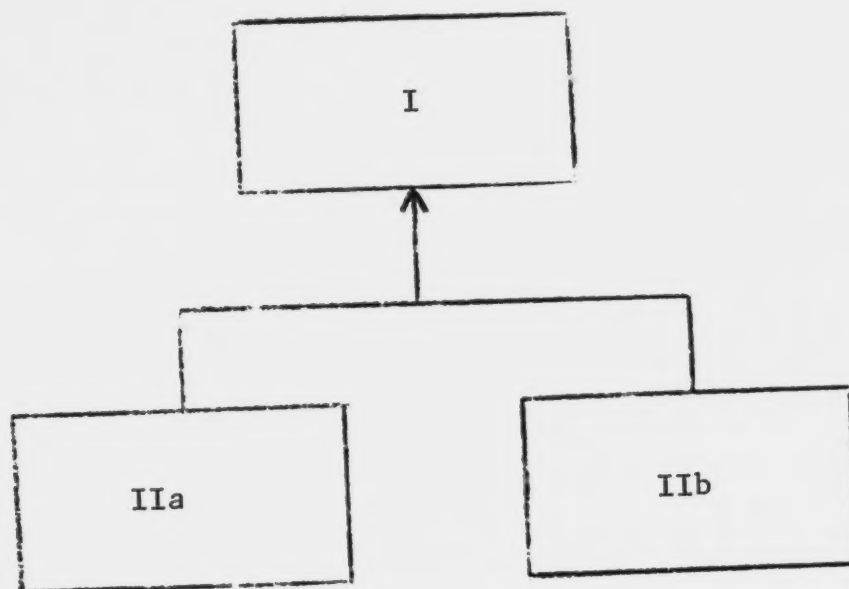
The manner by which behavioral objectives are written has been specified elsewhere (Mager, 1962; Tuckman, 1967) and will not be repeated here. Of relevance here is an explanation of why the hierarchy development described above utilized behavioral objectives. The answer is that competencies stated in behavioral terms can be more easily taught and evaluated. If you have decided on the behavior that the student is to manifest, then you can more readily determine the kinds of instructional experiences likely to lead to this behavioral outcome. Moreover, behavioral statements of competencies make it possible for evaluation materials to be developed so that students can evaluate their own progress, instructors can evaluate a student's progress, and curriculum developers can evaluate the success of instructional material. The advantages, thus, in using behavioral statements lie in the ease of subsequent instructional material development and evaluation. As will become apparent later, these advantages are considerable. The disadvantages of using such behavioral statements have been logically countered by Popham (1968).

²The further advantages of structural analysis as an aid to curriculum evaluation have been described by Tuckman (1967).

The Contingency Relationship

The hierarchies which result from structural analysis specify the level of requisite competencies and their connections. The successful attainment of any competency is contingent upon the successful attainment of all competencies that are subordinate to it (Gagne, 1962; Gagne and Paradise, 1961). Subordinate competencies are those at a lower level than the competency in question and connected to it by arrows. They are identified when the competency in question is analyzed into its pre-requisites.

Below is an example of a simple contingent relationship.



There are four possible outcomes that may be obtained when the performance of a student on the three competencies pictured above are measured subsequent to his having received instruction in all three areas. These outcomes are as follows:

	<u>Performance on IIa and IIb</u>	<u>Performance on I</u>
Outcome #1	+	+
Outcome #2	+	-
Outcome #3	-	-
Outcome #4	-	+
		+ correct; - incorrect

If outcome #1 is obtained, it indicates that instruction has been satisfactory in all three instances and the contingent relationship as pictured in the hierarchy is adequate (or at least that it is not obviously inadequate). If outcome #2 is obtained, it indicates that the additional instruction given to enable the student to progress from level II to level I has failed, or that there is another competency at level II upon which competency I is dependent, in addition to IIa and IIb, which has not been identified in the structural analysis (and, therefore, no instruction relevant to this unidentified competency has been included). Such an outcome should lead to further examination and testing to determine which of these two possibilities is correct. In terms of the hierarchy, outcome #2 indicates that, at worst, the hierarchy is incomplete. If outcome #3 is obtained, it indicates that instruction has been inadequate in teaching IIa and IIb, and that the contingent relationship may still be adequate. If outcome #4 is obtained, something is clearly wrong with the hierarchy. Outcome #4 indicates that the presumed relationship of contingency between IIa and IIb on the one hand and I on the other does not hold.

Because of the contingency relationship approach built into the

hierarchy, it is possible, after providing instruction and collecting data, to check on the adequacy of the hierarchy. (These ideas are also developed in Gagne and Paradise, 1961; Gagne, Mayor, Garstens, and Paradise, 1962; Tuckman, 1967.)

Designing Instructional Sequences

The first step in the preparation of instructional materials should be the preparation of the appropriate hierarchy. The hierarchy serves two important functions: (1) it tells the developer what to cover (thus, helping to avoid errors of omission); and (2) it guides the developer in sequencing his instructional material.

The hierarchy is a list of competencies which students must acquire en route to some terminal performance. In order for that terminal performance to be achieved, each and every competency uncovered by structural analysis and displayed in the hierarchy must be covered in sufficient depth in the instructional materials to insure its mastery by the students. Therefore, instructional materials specific to each competency in the hierarchy must be developed. Using the hierarchy to determine what to cover provides a greater guarantee for the developer that he will not leave anything out than would be true if he followed his "instincts". Thus, following the creation of the hierarchy, the developer must attempt to write materials for each competency at a sufficient level of detail that each can be mastered.

The hierarchy also suggests sequences in which the material may be ordered. Any competencies subordinate to another competency must be covered before covering the competency which is contingent upon their

successful mastery. In general, this means that competencies at lower levels are to be covered before those at higher levels. No necessary rules of ordering can be suggested for competencies at the same level. An example from the Fractions hierarchy may be helpful. Competency IIId, "subtracting 2 dissimilar fractions", has three direct subordinate competencies upon which it is contingent: IIAa ("supplying fractional equivalents"), IIAc ("identifying the LCD"), and IIIc ("subtracting 2 similar fractions"). Therefore, instruction on competencies IIAa, IIAc, and IIIc must precede instruction on competency IIId.

Since the hierarchy can be viewed as a set of contingencies between competencies subordinate to some final performance, we might say that the hierarchy provides for the possibility of contingency management. Contingency management has been referred to by Skinner (1968) in the context of "shaping" a student's behavior. Contingency management as used here would somewhat similarly indicate that the probability of occurrence of some desired end behavior would be increased by the arrangement of contingent competencies in an appropriate manner as specified by a hierarchy. Thus, the final end performance would be the result of prerequisite competencies added layer upon layer in accordance with the contingency relationships depicted in a hierarchy. The hierarchy would enable the curriculum developer to manage the contingencies upon which terminal performance is based by specifying what must be taught as prerequisites. Obviously, the procedure is a complex one since higher order prerequisites have prerequisites themselves as any hierarchy shows. However, these complex relationships

can be dealt with in instruction provided they have been identified. Structural analysis is the tool that provides for their identification.

Preparing Instructional Materials

The hierarchy answers the questions: What competencies are to be covered? and, In what order? The hierarchy does not prescribe the manner in which instructional materials are to be prepared to enable a student to master a competency beyond indicating its prerequisites. In other words, the hierarchy prescribes the design of instructional materials but not its preparation. Therefore, some comments are in order here about the preparation of instructional materials.

The approach to the preparation of instructional materials to be advocated here can be loosely described as programming. Programming refers to any approach which attempts to closely control and structure the activities of the student throughout the learning process. We can talk about programming the instructor as well as programming the instructional material. Programming requires a more systematic management of contingencies than is usually the case within instructional materials and instructional activities. The hierarchy provides for the specification and management of contingencies between competencies. However, some additional prescription is required to account for the activities of the student within competencies. Such a prescription is offered below:

In instructing a student in a particular competency, the sequence of instruction should take the following form:

- STEP 1. GOAL STATEMENT. A behavioral statement of the desired terminal performance of the competency will allow the student to know where he should end up and focus his learning on that goal.
- STEP 2. DEFINITIONS. Definitions which are unique to the competency in question must be covered. These may take the form of formulas, principles, concepts, etc.
- STEP 3. RECALL OF SUBORDINATE COMPETENCIES. When other competencies are subordinate, the student must be led to recall their content, and their relevance to the present goal must be indicated.
- STEP 4. INTEGRATION. Mastery of a competency requires that definitions and relevant prior material be integrated. Integrating instruction describes the integration procedure and its outcome.
- STEP 5. DEMONSTRATION. Within the integrating instruction, the sequence of steps required to demonstrate the particular competency in question is illustrated. It is then appropriate for the student to demonstrate that he is capable of carrying out these steps (i.e., manifesting the competency) with feedback.
- STEP 6. PRACTICE. After feedback on his demonstration, the student should be afforded the opportunity to practice the competency, with feedback.³

³The relative merits of practice as an aid to learning, recall, and transfer were the subject of studies by Gagne, Mayor, Garstens, and Paradise (1962), Tuckman (1962), and Tuckman et al. (1968).

A sequence of instructional materials illustrating the above six steps appears in the Appendix.

The progression of steps is based on principles of learning which have been identified and studied in the learning laboratory. Goal-directed behavior has been found to lead more quickly to a desired terminal performance than non-directed or incidental activity; therefore, the first step is the goal statement. Learning requires the integration of new material with relevant prior knowledge; thus, new material is presented, the recall of prior learning stimulated, and integrating instructions provided. Since performance depends on the generation of a sequence of behaviors, integrating instructions which illustrate that sequence are essential. Following these instructions, it is time for the learner to perform. Research has shown that the participation of the learner in the learning process can facilitate attention by keeping the learner alert; can contribute to the replacement of incorrect response tendencies with correct ones - through the mechanism of feedback; and can provide motivation through the maximization of success over failure - the outcome resulting from the control of difficulty level of required performances and from good prior instruction. Once the learner has demonstrated an accurate performance, practice has been shown to further insure its establishment. Furthermore, transfer can be facilitated by including practice experiences that utilize the same principles or concepts covered in the instructional sequence but vary in their particulars. Thus, the student will learn to apply the concepts learned in the instructional sequence in a variety of appropriate situations.

Summary

This paper has described a systematic approach to curriculum development called structural analysis which represents an attempt to analyze terminal performance objectives for a unit of subject matter into a sequence of subordinate or prerequisite competencies which must be satisfactorily mastered if successful terminal performance is to occur. The technique for identifying these competencies is to progressively ask the question: What competencies must a person already possess in order to attain a satisfactory performance level on some specified objective, given no instruction beyond those definitions specific to the objective in question? The result of asking this question of all competencies identified as a result of the application of this technique is the generation of a hierarchy of requisite competencies which, ideally, parallels the learning process appropriate to the final task.

Competencies are arranged in the hierarchy by level, going from complex to simple. Successful attainment of any competency in the hierarchy is theoretically contingent upon successful attainment of all competencies prerequisite to it. In preparing instructional materials, the hierarchy provides the developer with a guide of what to cover and the sequence to follow. Because the hierarchy specifies the contingent relationship among competencies, it provides the curriculum developer with sufficient information to engage in contingency management. By following the sequence illustrated in the hierarchy, those contingencies appropriate to final task success will be met and the probability of successful performance maximized. Thus, the hierarchy describes what to cover and in what order.

In preparing instructional materials appropriate for a single specific competency, the following progression of instruction-relevant activities were recommended: (1) goal statement, (2) definitions, (3) recall of prior knowledge, (4) integration, (5) demonstration, (6) practice. The use of this approach is equivalent to programming; lessons given by teachers as well as by text materials can be so programmed. Within the above sequence, it is advantageous to require that the learner participate in the learning process by responding. The use of immediate feedback provides for the more effective management of the learning process.

Thus, structural analysis leading to the depiction of competencies prerequisite to the performance of some final objective in an hierarchical array, coupled with the within-competency instructional sequence stated above, provides the curriculum developer with a valuable tool for systematically developing instructional materials that effectively lead to a desired behavior. Such an effective arrangement of competencies based on their contingent relationship further requires the consistent use of behavioral statements in the description of desired-end states and their prerequisite competencies. The use of the techniques advocated in this paper, while time-consuming, will insure that instructional materials so developed will bear more than a casual and fortuitous relation to the learning process.

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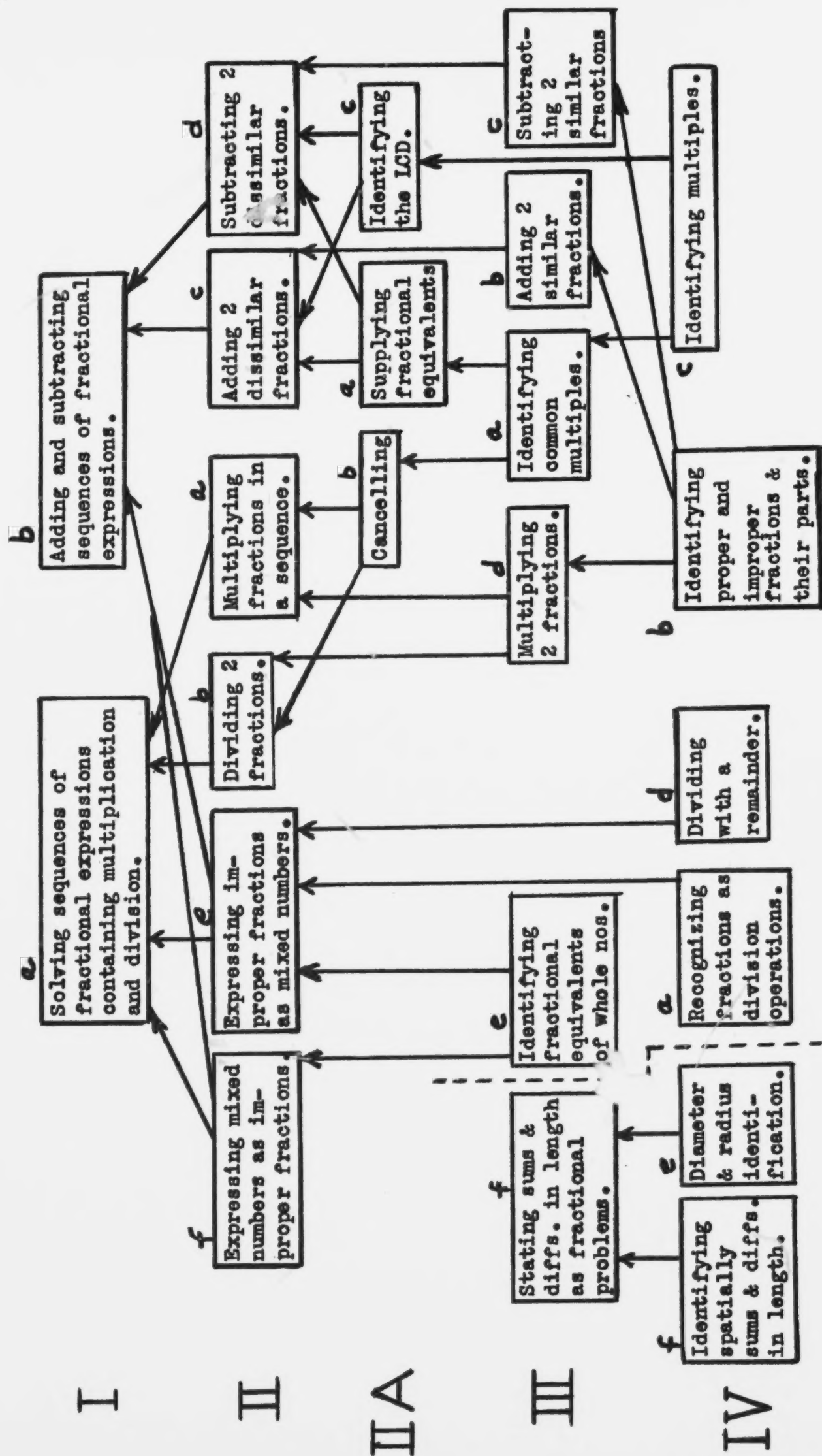
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APPENDIX

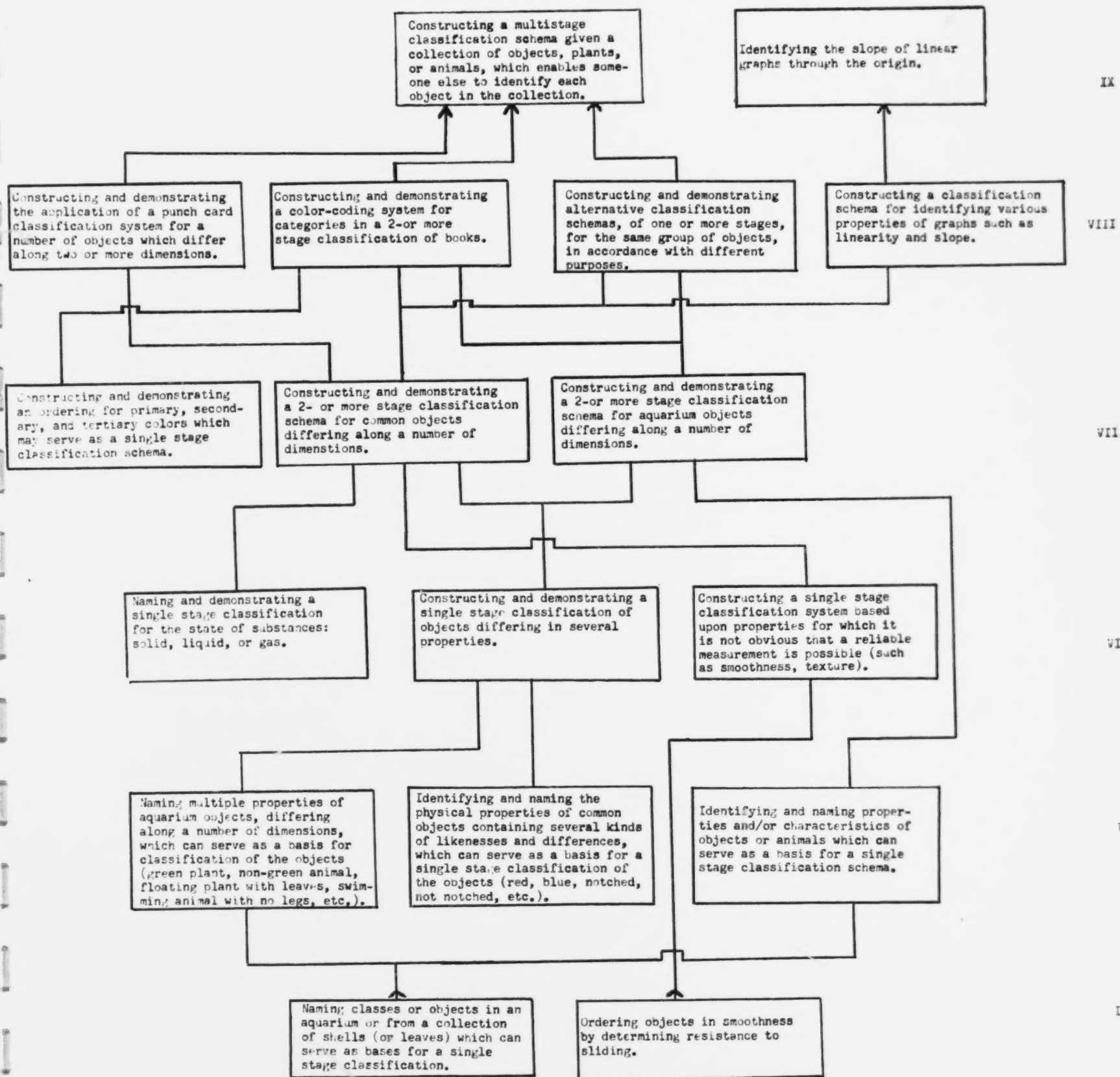
FRACTIONS

HIERARCHY FOR PERFORMING OPERATIONS ON FRACTIONS (Tuckman, 1968)

Task: finding single numerical values for expressions containing dissimilar fractional expressions in sequence (either taken from drawing dimensions or stated directly) requiring addition, subtraction, multiplication, and division.



HIERARCHY FOR CLASSIFICATION IN ELEMENTARY SCIENCE (AAAS, 1965)



HIERARCHY FOR ADDING IN THE "NEW MATHEMATICS"
(Gagne, Mayor, Garstens, and Paradise, 1962)

TASK 1

TASK 2

STATING, USING SPECIFIC NUMBERS, THE SERIES OF STEPS NECESSARY TO FORMULATE A DEFINITION OF ADDITION OF INTEGERS, USING WHATEVER PROPERTIES ARE NEEDED, ASSUMING THOSE NOT PREVIOUSLY ESTABLISHED

ADDING INTEGERS

Ia
SUPPLYING THE STEPS AND IDENTIFYING THE PROPERTIES ASSUMED IN ASSERTING THE TRUTH OF STATEMENTS INVOLVING THE ADDITION OF INTEGERS

Ib
STATING AND USING THE DEFINITION OF THE SUM OF TWO INTEGERS, IF AT LEAST ONE ADDEND IS A NEGATIVE INTEGER

IIa
SUPPLYING OTHER NAMES FOR POSITIVE INTEGERS IN STATEMENTS OF EQUALITY

IIb
IDENTIFYING AND USING THE PROPERTIES THAT MUST BE ASSUMED IN ASSERTING THE TRUTH OF STATEMENTS OF EQUALITY IN ADDITION OF INTEGERS

IIIa
STATING AND USING THE DEFINITION OF ADDITION OF AN INTEGER AND ITS ADDITIVE INVERSE

IIIb
STATING AND USING THE DEFINITION OF ADDITION OF TWO POSITIVE INTEGERS

IVa
USING THE WHOLE NUMBER 0 AS THE ADDITIVE IDENTITY

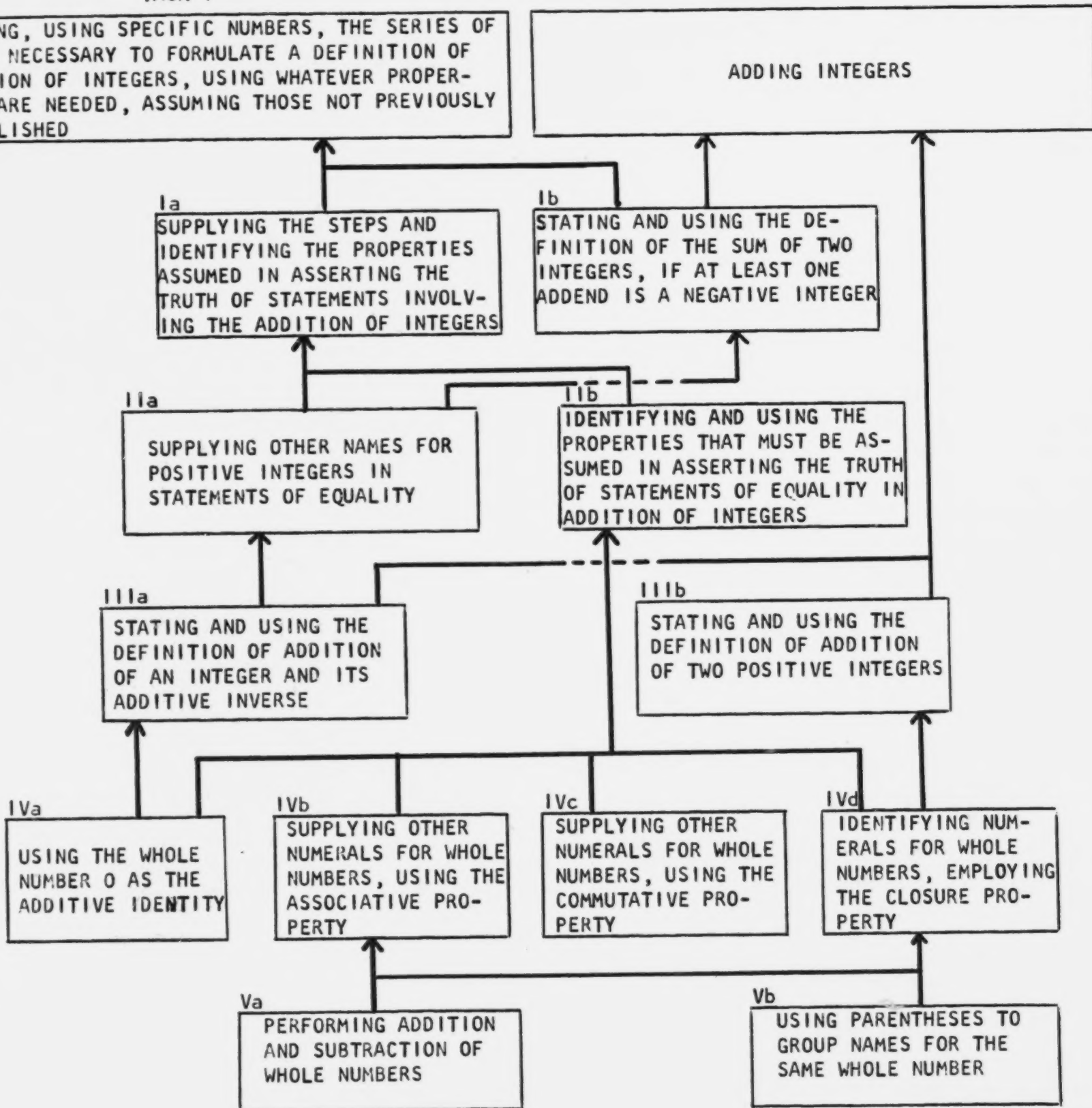
IVb
SUPPLYING OTHER NUMERALS FOR WHOLE NUMBERS, USING THE ASSOCIATIVE PROPERTY

IVc
SUPPLYING OTHER NUMERALS FOR WHOLE NUMBERS, USING THE COMMUTATIVE PROPERTY

IVd
IDENTIFYING NUMERALS FOR WHOLE NUMBERS, EMPLOYING THE CLOSURE PROPERTY

Va
PERFORMING ADDITION AND SUBTRACTION OF WHOLE NUMBERS

Vb
USING PARENTHESES TO GROUP NAMES FOR THE SAME WHOLE NUMBER



Sequence of Frames from Performing Operations on Fractions
(Tuckman, 1968). Competency IIc: Adding Dissimilar Fractions

1. Now you are going to learn how to add two dissimilar fractions, that is, fractions with different denominators.	
2. Two dissimilar fractions are added by finding the LCD (lowest common denominator); changing the 2 fractions into equivalent fractions with the LCD as the denominator (thus making the 2 dissimilar fractions similar); and then adding the numerators of the 2 resulting similar fractions.	
3. To accomplish addition of 2 dissimilar fractions you must remember how to find the LCD. Three denominators: 3, 5, 6 LCD = _____ You must also remember how to change one fraction into another equal fraction with a different denominator. Change $\frac{2}{5}$ into 25th's. _____	
4. Finally, you must remember how to add 2 similar fractions (and to reduce the sum to lowest terms). $\frac{3}{14} + \frac{4}{14} =$ _____	Ans. to 3 LCD = 30 $\frac{10}{25}$
5. Now combine all these ideas in the following manner: If you are asked to add $\frac{1}{2} + \frac{2}{3}$ you do the following: first you want to change these dissimilar fractions into _____ common fractions which you know how to add.	Ans. to 4 $\frac{7}{14} = \frac{1}{2}$
6. To change $\frac{1}{2}$ and $\frac{2}{3}$ into similar fractions you must first find the LCD. In this case, the LCD = _____	Ans. to 5 similar

7. Now that you have determined that the LCD is 6, you must change $\frac{1}{2}$ and $\frac{2}{3}$ into 6th's.

$\frac{1}{2}$ changed into 6th's is $\frac{3}{6}$

$\frac{2}{3}$ changed into 6th's is _____

Ans. to 6

6

8. The problem of adding the 2 dissimilar fractions $\frac{1}{2}$ and $\frac{2}{3}$ has been changed into the equivalent problem of adding the 2 similar fractions $\frac{3}{6}$ and $\frac{4}{6}$. Now solve this new addition:

$$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \underline{\hspace{2cm}}$$

Ans. to 7

$\frac{4}{6}$

9. $\frac{1}{3} + \frac{3}{5} = \underline{\hspace{2cm}}$

Ans. to 8

$\frac{7}{6}$

10. $\frac{3}{4} + \frac{1}{6} = \underline{\hspace{2cm}}$

Ans. to 9

$\frac{14}{15}$

11. $\frac{1}{6} + \frac{1}{8} = \underline{\hspace{2cm}}$

Ans. to 10

$\frac{11}{12}$

Ans. to 11

$\frac{7}{24}$

Frame 1
Frame 2
Frames 3, 4
Frames 5,6,7,8
Frames 9,10,11

Goal Statement
Definitions
Recall of Prior Learning
Combination of Integrating
Instruction and Demonstration
Practice